
DISSIPATIVE MODELS IN PHASE TRANSITIONS
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ABSTRACTS BOOK

 ONE-DIMENSIONAL SHAPE MEMORY ALLOY PROBLEM WITH SMALL VISCOSITY

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This is a joint work with Prof. A. Kadoya, Hiroshima-Shudo University, Japan and S. Yoshikawa, Tohoku University, Japan. In this talk we consider the following system: To find the displacement $u = u(t, x)$, the stress $\sigma = \sigma(t, x)$ and the temperature field $\theta = \theta(t, x)$ on $Q(T) := (0, T) \times (0, 1)$ satisfying

$$u_{tt} + \gamma u_{xxxx} - \mu u_{xxt} = \sigma_x \quad \text{in } Q(T), \quad (1)$$

$$\theta_t - \kappa \theta_{xx} = \mu |u_{xt}|^2 + \sigma u_{xt} \quad \text{in } Q(T), \quad (2)$$

$$\sigma_t + \partial I(\theta, \varepsilon; \sigma) \ni c u_{xt} \quad \text{in } Q(T), \quad (3)$$

$$u(t, 0) = u(t, 1) = 0 \text{ and } u_{xx}(t, 0) = u_{xx}(t, 1) = 0 \quad \text{for } 0 < t < T, \quad (4)$$

$$\theta_x(t, 0) = \theta_x(t, 1) = 0 \quad \text{for } 0 < t < T, \quad (5)$$

$$\sigma_x(t, 0) = \sigma_x(t, 1) = 0 \quad \text{for } 0 < t < T, \quad (6)$$

$$u(0) = u_0, u_t(0) = v_0, \theta(0) = \theta_0, \sigma(0) = \sigma_0 \quad \text{on } (0, 1), \quad (7)$$

where the strain $\varepsilon = u_x$, γ , μ , κ and c are positive constants and u_0 , v_0 , θ_0 and σ_0 are initial functions. Moreover, I is the indicator function of the interval $[f_*(\theta, \varepsilon), f^*(\theta, \varepsilon)]$. The above system (1) ~ (7) is the mathematical model for the dynamics of the one-dimensional shape memory alloy material. The kinetic equation leads to (1), and (2) is the heat equation. Here, we assume the viscosity for the stress. For the shape memory alloy material the relationship between the stress σ and the strain ε is described by the hysteresis operator. In this talk we assume that the relationship is the generalized stop operator, which is characterized by the ordinary differential equation including the subdifferential of the indicator function. Now, f_* and f^* are corresponding to the lower and upper curves, respectively, of the hysteresis loop.

In [1] we have proved the existence and the uniqueness of a solution of the above system under the restriction for data, $\mu^2 > 4\gamma$. However, from experiment results it is known that μ is less than γ so that the assumption is not good. In this talk we show the well-posedness of the problem for any $\mu > 0$ and $\gamma > 0$. The key of the proof is an estimate for the kernel of the complex Landau-Ginzburg equation.

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EXTREME ANISOTROPY OF DIFFUSE INTERFACES IN CRYSTALS

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We will begin by discussing the basics on crystals, and anisotropic surface tension. After introducing a model of anisotropic diffuse interfaces, we focus on extreme anisotropy. We investigate certain plane wave solutions and their stability. These plane wave solutions correspond to planar interfaces, that is to various planar cuts of the crystal. Different orientations of these planes with respect to the crystal axes give rise to different surface energies. Mathematically we have to handle systems of two 2nd order equations, one of which involves a singular parameter that measures the anisotropy. The solutions that are constructed can be thought as the analogs of the familiar heteroclinic connection for the bistable equation.

SHAPE MEMORY ALLOYS: CONSTITUTIVE MODELING, SOLUTION ALGORITHMS
AND NUMERICAL SIMULATIONS

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An always increasing knowledge on material properties as well as a progressively more sophisticated production technology make Shape Memory Alloys (SMA) extremely interesting for the industrial world. At the same time, SMA devices are typically characterized by complex multi-axial stress states as well as non-homogeneous and non-isothermal conditions both in space and time.

This aspect suggests the finite element method as a useful tool to help and improve application design and realization. With this aim, we focus on three-dimensional macroscopic thermo-mechanical modeling able to reproduce the most significant SMA features, such as pseudo elasticity and shape memory effect. Moreover, for the proposed models it is possible the development of a time-discrete solution algorithm, which is effective and robust. We verify the computational tool ability to simulate complex pointwise stress-strain histories as well as realistic mechanical boundary value problems.

As an example we may consider coronaric stents, which are small metal tubes, inserted into an artery at the site of a narrowing to act as an internal scaffolding or support to the blood vessel. Other simulations and details can be found on the web page http://www.unipv.it/dms/auricchio/cofin_02/home.htm

FINITE ELEMENT APPROXIMATION OF A PHASE FIELD MODEL FOR THE
ELECTROMIGRATION OF INTERGRANULAR VOIDS IN A STRESSED INTERCONNECT

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In [1], we considered a fully practical finite element approximation of the nonlinear degenerate parabolic system: Find $\{u(x, t), w(x, t), \phi(x, t)\}$ s.t.

$$\gamma \frac{\partial u}{\partial t} = \nabla \cdot (b(u) \nabla [w + \alpha \phi]), \quad w = -\gamma \Delta u + \gamma^{-1} \Psi'(u), \quad \nabla \cdot (c(u) \nabla \phi) = 0.$$

Here $\gamma \in \mathbb{R}_{>0}$ is the interfacial parameter, $\alpha \in \mathbb{R}$, Ψ is a nonsmooth double well potential, and $c(u) := 1 + u$, $b(u) := 1 - u^2$ are degenerate coefficients. The degeneracy in b restricts $u \in [-1, 1]$. The above, in the limit $\gamma \rightarrow 0$, models the evolution of voids by surface diffusion and electromigration, if $\alpha \neq 0$, in an electrically conducting solid. In [2] and [3] we extend this phase field model to include the effects of (i) stressmigration and (ii) grain boundaries.

In addition to showing stability bounds for our finite element approximations, we prove convergence as the mesh parameters go to zero, and hence existence of a solution to these nonlinear degenerate parabolic systems for fixed γ in two space dimensions. Furthermore, iterative schemes for solving the resulting nonlinear discrete systems are introduced and analyzed. Finally, some numerical experiments are presented.

Parts in collaboration with

Harald Garcke (Regensburg, Germany): Stressmigration/Intergranular.

Vanessa Styles (Sussex, UK): Electromigration.

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NONLOCAL EVOLUTION EQUATIONS - BASIC THEORY AND WAVES

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We consider various lattice dynamical systems with long range interaction and related integro-differential evolution equations.

The Nonlocal Allen-Cahn equation has the form

$$u_t = d(J * u - u \int_{\Omega} J) + f(u),$$

where $d > 0$, f is bistable, and $J * u \equiv \int_{\Omega} J(x - y)u(y)dy$. (The discrete version is also considered.) This is studied for the ferromagnetic case when the kernel J is nonnegative and also for the more difficult case when J may change sign. When $\Omega = \mathbb{R}$ we give conditions under which traveling waves exist, even when $J(x)$ changes sign with x , a situation usually associated with pattern formation. This is joint work with A. Chmaj and X. Chen.

For the Nonlocal Cahn-Hilliard equation,

$$u_t = -\Delta(d(J * u - u \int_{\Omega} J) + f(u)),$$

with either homogeneous Neumann or Dirichlet boundary conditions, we establish the basic well-posedness for the initial value problem and provide some results on long-term behavior. Additional results along these lines are given for a Nonlocal Phase-Field System. This is joint work with J. Han and G. Zhao.

Finally, if time permits, I will discuss joint work with C. Zhang in which we obtain traveling pulse solutions to a nonlocal wave equation,

$$u_{tt} = d(J * u - u \int_{\Omega} J) + f(u).$$

WELL-POSEDNESS RESULTS FOR A MODEL OF DAMAGE
IN THERMOVISCOELASTIC SYSTEMS

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We deal with a PDE's system describing damage in thermoviscoelastic materials. Referring to the modelling approach due to M. Frémond [F], we derive the evolution equations of the system by basic balance laws of continuum mechanics coupled with the principle of virtual power and we write the constitutive relations by a free energy functional and a pseudo-potential of dissipation.

We choose as state variables of the model the absolute temperature θ , the strain tensor $\varepsilon(\mathbf{u})$ (\mathbf{u} stands for the vector of small displacements), a phase parameter χ related to the quantity of the damaged material, and the gradient of the damage $\nabla\chi$, accounting for local interactions. Hence, in this approach, the process of damage is modelled as a dissipative phase transition phenomenon. Internal constraints are assumed on the phase variable and on its time derivative: χ must belong to $[0, 1]$ ($\chi = 1$ and $\chi = 0$ correspond to the undamaged material and to the completely damaged system, respectively) and χ_t is forced to be non-positive, in order to account for the irreversibility of the damaging process. In the sequel, we will let \mathbf{u} be a scalar quantity u .

Hence, letting the material be located in a smooth bounded domain $\Omega \subset \mathbf{R}^3$ up to a given time $T > 0$, we describe the evolution of the damage by the following system

$$\theta_t - \Delta\theta = \theta\chi\nabla u_t \cdot \mathbf{a} + \theta\chi_t\nabla u \cdot \mathbf{a} + |\chi_t|^2 + |\nabla\chi_t|^2 + |\nabla u_t|^2 + r, \quad (1)$$

$$\chi_t - \Delta\chi_t - \Delta\chi + \partial I_{(-\infty, 0]}(\chi_t) + \partial I_{[0, 1]}(\chi) \ni w - \frac{1}{2}|\nabla u|^2 - \theta\nabla u \cdot \mathbf{a}, \quad (2)$$

$$u_{tt} - \Delta u_t - \operatorname{div}(\chi\nabla u + \chi\theta\mathbf{a}) = f \quad \text{a.e. in } \Omega \times]0, T[, \quad (3)$$

where the positive coefficient w stands for the cohesion energy of the material, \mathbf{a} is a constant vector related to the thermal expansion coefficient, r represents an external heat source and f an external volume force applied to the structure; finally, $\partial I_{(-\infty, 0]}$ and $\partial I_{[0, 1]}$ denote the subdifferentials of the indicator functions on the intervals $(-\infty, 0]$ and $[0, 1]$, respectively.

We remark that in (1) we deal with the *complete* energy balance equation, as we retain *all* the mechanically induced heat sources (the dissipative quadratic terms at the right hand side), even if they are high order nonlinearities.

For the system (1)-(3), complemented by an appropriate set of initial and boundary conditions, we prove in [BB] a local-in-time existence result, performed by a fixed point procedure; moreover, we establish the uniqueness result, by some global contracting estimates.

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SUPER AND ULTRA CONTRACTIVITY FOR DOUBLY NONLINEAR
EVOLUTION EQUATIONS

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We use logarithmic Sobolev inequalities involving the p -energy functional to prove L^p - L^q smoothing and decay properties, of supercontractive and ultracontractive type, for the semigroups associated to doubly nonlinear evolution equations of the form $\dot{u} = \Delta_p(u^m)$ (with $(m(p-1) \geq 1)$ in an arbitrary Euclidean domain, homogeneous Dirichlet boundary conditions being assumed. This class of evolution equations contains as special cases the porous media equation (take $p = 2$) or the p -Laplacian equation (take $m = 1$) and the linear heat equation (take $m = 1$ and $p = 2$) as well. The bounds are of the form $\|u(t)\|_q \leq C \|u_0\|_r^\gamma / t^\beta$ for any $r \leq q \in [1, +\infty]$ and $t > 0$ and the exponents β, γ are explicitly calculated and are shown to be the only possible for a bound of such type. Since the techniques used are essentially functional analytic, the bounds also hold in more general situations such as in the case of the filtration equation $\dot{u} = \Delta \varphi(u)$, with some technical assumptions on the real valued function φ . A generalization to the Riemannian manifolds setup is also considered.

AN EVOLUTION PROBLEM IN A MATERIAL WITH MEMORY

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An evolution problem in the framework of linear thermodynamics is considered. Specifically, the temperatures' time derivative turns out to be related to the the heat flux gradient in a material with memory. The problem to be studied is a partial differential

one in which the unknown is represented by the temperature distribution, in space and time, in a rigid heat conductor with memory. The initial spacial temperature distribution is given and, in addition, the condition of no heat flux through the boundary of the conductor is assumed.

GEOMETRICAL IMAGE SEGMENTATION BY THE ALLEN-CAHN EQUATION

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Pattern recovery (or image segmentation) is a subtask of image processing whose goal is to extract pattern boundaries from an original, possibly noisy, image. We present an algorithm of pattern recovery based on the solution of the modified Allen-Cahn equation. This modification is new among the phase-field models. The approach is usually understood as a regularization of the levelset motion by mean curvature where we impose a special forcing term which lets the initial levelset closely surround the pattern in question. We show convergence of the fully discrete numerical scheme and demonstrate function of the algorithm on several artificial as well as real images.

SOME RECENT RESULTS ON PARABOLIC INVERSE PROBLEMS
FOR PHASE-FIELD MODELS WITH MEMORY

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The literature related to parabolic inverse problems with memory is very wide, but only recently identification inverse problems for phase-field models have been studied. In this talk we will introduce two types of phase-field models of parabolic type and we will present some related inverse problems. Finally, we show some recent results and the methods to obtain them.

ON THE BECKER DÖRING SYSTEM AND RELATED ITEMS IN THE PRESENCE OF
SURFACE TENSION AND MULTIAXIAL STRESSES

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Phase transitions in solids are usually induced, or at least strongly influenced by surface tension and multiaxial stresses which give rise to nonzero stress deviators. We consider the following important example:

Nucleation of liquid droplets in semi-insulating GaAs is accompanied by deviatoric stresses resulting from the liquid/solid misfit. A competition of surface tension and the stress deviator at the interface determines the nucleation barrier. Under some circumstances the stress deviator may decrease the nucleation barrier to zero.

The central quantity for the description of all aspects of phase transitions is the chemical potential, which can be additively decomposed into a chemical and a mechanical part. In particular the calculation of the mechanical part of the chemical potential is of crucial importance. We determine the chemical potential in the framework of the Kirchhoff–St. Venant law which gives an appropriate stress/strain relation for many solids in the small strain regime. We establish criteria, that allow to replace the Kirchhoff–St. Venant law the simpler Hooke law.

The evolution of liquid droplets in semi-insulating GaAs is due to diffusional processes in the vicinity of the droplet. The diffusion flux results from a competition of chemical and mechanical driving forces. The evolution of the size distribution of the droplets is determined by the Becker Döring system. The study of its properties in the presence of nonzero deviatoric stresses is the main subject of this lecture.

FORMATION OF SOLID DEPOSITS IN WAXY CRUDE OIL

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Oils with a sufficiently high content of heavy hydrocarbons (waxy crude oils) have a tendency of developing solid wax deposit in pipelines crossing cold regions. The responsible mechanism is radial diffusion of dissolved wax induced by thermal gradient. The phenomenon of thermally induced mass transport in saturated solutions is studied and applications are presented to the measurement of wax diffusivity and to the

prediction of the deposit growth in a pipeline. Work performed in cooperation with EniTecnologie (Milano).

MECHANICS AND PHASE CHANGE

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The classical ice-water phase change involves microscopic motions which have macroscopic effects. In shape-memory alloys the solid-solid phase change involves also microscopic motions which are responsible for phase twinning. When damaging structures, microscopic motions break the links which are responsible for the cohesion of the material. We think that these motions have to be taken into account in the macroscopic predictive theories. The basic idea we have developed is to account for the power of the microscopic motions in the power of the interior forces. Thus we modify the expression of the power of the interior forces and assume that it depends on the volume fraction rate, micro-voids volume fractions rate, damage rate, ... which are clearly related to the microscopic motions. Furthermore we assume that it depends also on the gradient of these rates to account for local microscopic interactions. The consequences of this assumption give the basic equations which may be applied in numerous predictive theories: phase change problems, damage, shape memory alloys, ... Moreover when phase change occur, temperatures may be discontinuous, for instance when warm rain falls on frozen soil.

The mechanical predictive theories result in partial differential equations which are investigated with mathematical analysis. We give examples of the interaction of mathematics and mechanics to provide models which may be used in engineering.

STEFAN PROBLEM WITH CONVECTION GOVERNED BY NAVIER-STOKES EQUATION

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The Czochralski pulling method of crystal growth process is widely used for the production of a column of single silicon crystal from the melt. This process has many

interesting physical aspects; for instance, there is neither phase transition between gas and liquid nor between gas and solid, and the solid-liquid phase transition occurs on its interface, which means that the balance equation on interface is of the Stefan type. In the mathematical modeling due to the paper of Pawłow 2002, the solidification of melt substances yields a smooth change of the material configuration in time and a smooth flow in the material. Therefore the shape of the material domain $\Omega_m(t)$ changes smoothly in time. Our idea is to describe this phenomenon as a Stefan problem with convection. The case when the convective vector \mathbf{v} in the liquid domain is unknown, too, it will be found as a part of solution of our problem. Our model is a enthalpy method of the Stefan problem formulated in a non-cylindrical domain

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u - \Delta \theta = f \quad \text{in } Q_m := \bigcup_{t \in (0, T)} \{t\} \times \Omega_m(t),$$

where θ is temperature, u is enthalpy. We assume that the convection \mathbf{v} is governed by the Navier-Stokes equation in the unknown liquid region $Q_\ell(u)$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \nu_\ell \Delta \mathbf{v} + \nabla p_\ell = \mathbf{g}_\ell(\theta) \quad \text{in } Q_\ell(u),$$

with $\operatorname{div} \mathbf{v} = 0$ in $Q_\ell(u)$. In the unknown solid region $\mathbf{v} = \mathbf{v}_D$ in $Q_s(u)$. The main difficulty in our mathematical treatment arises from the time-dependence of the material domain and lack of regularity properties of enthalpy and convection.

TRIPLE JUNCTIONS IN FOURTH ORDER GEOMETRIC EVOLUTION EQUATIONS

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In certain cases diffusion in a multicomponent alloy is restricted to interfaces between different phases. This is called surface diffusion and it leads to a geometric evolution law in which the normal velocity of the evolving interface is given by the negative Laplacian of the mean curvature. In the case of more than two phases, interfaces might meet at triple junctions and in this case boundary conditions, e.g. determining angle conditions or flux balances, have to hold at triple junctions. The main driving force of the diffusion process is the reduction of interfacial area and during the evolution the volume fraction of the phases is preserved.

It has been shown by Garcke and Novick-Cohen that the above geometric evolution law can be derived from a Cahn-Hilliard system with degenerate mobility. In this talk we review the derivation of the geometric evolution equation and we study the stability of stationary solutions.

 MATHEMATICAL STUDY OF A PHASE TRANSITION MODEL

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A phase field model for phase transition based on entropy balance is stated in its mathematical form. The state variables are the absolute temperature and an order parameter. While the equation that rules the phase dynamics is quite standard, the equation for the temperature is integrodifferential and singular. Indeed, a laplacian acting on a convolution term for the temperature appears; on the other hand, the time derivative is applied to the logarithm of the temperature. The talk deals with the mathematical analysis of an initial-boundary value problem for such a system and presents the results obtained in joint works with E. Bonetti, P. Colli, and M. Fabrizio.

 PHASE-FIELD MODELS FOR MATERIALS WITH MEMORY
 AND APPLICATIONS TO GLASS TRANSITION

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Many efforts have been devoted to formulate "a thermodynamically consistent" phase-field model, especially in connection with thermally-induced reversible phase transitions. In particular, we quote the works by Penrose & Fife [1], Fried & Gurtin [2] and Frémond [3]. Mainly, Penrose & Fife consider a framework based completely on relaxation laws with entropy as potential, Gurtin & Fried make a systematic use of general balance laws, according to modern continuum thermodynamics, Frémond's approach looks like a mix of both. However, the crucial point is the different expression they consider for the first law of thermodynamics. Assuming, as usual, a negligible contribution of the macroscopic stress power, in the papers by Penrose & Fife the energy balance equation involves the heat flux, only:

$$(1) \quad \rho \dot{e} = -\nabla \cdot \mathbf{q} + \rho r.$$

On the contrary, an "extra energy flux" is present in the other approaches, namely

$$\rho \dot{e} = -\nabla \cdot \mathbf{q} + \nabla \cdot \mathbf{h} + \rho r,$$

where \mathbf{h} is named "power of accretive stresses" (Gurtin & Fried) or "power of microscopic forces" (Frémond). According to [2], this suggestion traces back to S.-K.Chan. It is worth noting that all of them gives the classical form of the standard model (often referred as *Caginalp model*) after linearization with respect to temperature in a neighborhood of the transition value θ_0 (see [2,4]).

The first aim of the present work is to establish a different approach based on the introduction of an "extra entropy flux" \mathbf{k} , due to the phase-field evolution, into the Clausius–Duhem entropy inequality

$$(2) \quad \rho \dot{\eta} + \nabla \cdot \left(\frac{\mathbf{q}}{\theta} \right) + \nabla \cdot \mathbf{k} - \frac{\rho r}{\theta} \geq 0,$$

where $\mathbf{k} = \dot{\varphi} \boldsymbol{\xi}$, $\varphi \in [-1, 1]$ being the phase parameter. On the other hand, the energy balance law is unaffected by "extra energy fluxes" and takes the form (1). Combining (1) and (2) we derive the *dissipation inequality*

$$(3) \quad -\rho(\dot{\psi} + \eta \dot{\theta}) - \frac{1}{\theta} \mathbf{q} \cdot \nabla \theta + \theta \nabla \cdot \mathbf{k} \geq 0,$$

which is used to restrict constitutive relations for ψ , η , \mathbf{q} , $\boldsymbol{\xi}$, etc. The evolution law for the phase-field (the so called *kinetic* or *Ginzburg–Landau equation*) is then obtained by making a special assumption which reads either as a *supplementary balance law*, namely

$$(A) \quad \nabla \cdot \boldsymbol{\xi} = \pi + \nu \dot{\varphi}, \quad \nu > 0,$$

or as a *constitutive relation*

$$(B) \quad \dot{\varphi} = f,$$

where φ is regarded as an internal variable. In any case, the exploitation of (3) suggests the (not unique!) choice of the constitutive functions f , $\boldsymbol{\xi}$, π , ν , etc.

In the framework of continuum thermodynamics, this procedure allows us to recover, as a special case, the phase-field system proposed by Penrose & Fife for first- and second-order transitions.

The second aim of this work is to construct a general theory of phase transitions in materials with memory. This can be done using the previous analysis as a basis and allowing constitutive functionals to depend on the so called "summed past histories" of temperature and temperature gradient, respectively

$$\zeta^t(\tau) = \int_0^\tau \theta(t-s) ds, \quad \mathbf{g}^t(\tau) = \int_0^\tau \nabla \theta(t-s) ds, \quad t > \tau \geq 0.$$

So doing, we have a proper starting point to describe glass transition in liquids of high viscosity (glycerol, for instance). In fact, as pointed out by Jackle [5], liquids of this kind exhibit thermal memory effects at temperatures near the glass transition value. Since this is a second-order liquid–solid phase transition, the theory proposed in a recent unpublished report [6] and exposed at PhTIEE (S.Margherita Ligure, 2000) does not apply to this case.

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ORDER - DISORDER TRANSITION IN ALLOYS

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In 1992, Cahn and Novick-Cohen have enlarged the classical Cahn-Hilliard model of phase separation to a system of two equations, supposed to model simultaneously phase separation and an order-disorder transition. I will present an analysis of this model and results of numerical computations, which give an interesting intuition about the attractor structure for the system.

MEMORY RELAXATION OF ALLEN-CAHN AND CAHN-HILLIARD EQUATIONS

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A reaction-diffusion type equation is relaxed by means of a time convolution operator, with a kernel obtained rescaling a given positive decreasing function. This relaxation produces an integrodifferential equation, whose formal limit, as the scaling parameter (or relaxation time) tends to zero, is the original equation. In particular, if the memory kernel is the decreasing exponential, then the relaxed equation is equivalent to the widely studied hyperbolic relaxation. We present quite general conditions which ensure that the longterm dynamics of the two evolution equations are close, in some appropriate sense, when the relaxation time is small. More precisely, we show the existence of a family of robust exponential attractors for the related dissipative dynamical systems, which is stable with respect to the relaxation time. This abstract theorem is then applied to Allen-Cahn and Cahn-Hilliard equations. The results are part of a joint research project with S. Gatti, A. Miranville, and V. Pata.

ON A THERMOMECHANICAL MODEL OF PHASE TRANSITIONS IN STEEL

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In the talk I will discuss a macroscopic thermomechanical model of phase transitions. Effects like transformation strain and transformation plasticity induced by the phase transitions will be considered and used to formulate a consistent thermomechanical model. The physical parameters are allowed to depend on the respective phase volume fractions by a mixture ansatz.

The resulting system of state equations consists of a quasistatic momentum balance coupled with a nonlinear stress-strain relation, a nonlinear energy balance equation and a system of ODEs for the phase volume fractions.

I will prove the existence of a weak solution and conclude with some results of numerical simulations.

TWO-PHASE FREE BOUNDARY PROBLEM WITH KINETIC CONDITION

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We prove existence of a unique classical solution for a two-phase Stefan problem with kinetic for continuous initial and boundary conditions.

ASYMPTOTIC BEHAVIOUR FOR A PHASE-FIELD MODEL WITH HYSTERESIS
IN THERMO-VISCO-PLASTICITY

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As a mathematical model for nonlinear thermo-visco-plastic developments the following system has been derived in [3]:

$$\rho u_{tt} - \mu u_{xxt} = \sigma_x + f, \quad (8)$$

$$\sigma = \mathcal{H}_1[u_x, w] + \theta \mathcal{H}_2[u_x, w], \quad (9)$$

$$(C_V \theta + \mathcal{F}_1[u_x, w])_t - \kappa \theta_{xx} = \mu u_{xt}^2 + \sigma u_{xt} + g, \quad (10)$$

$$\nu w_t + \mathcal{H}_3[u_x, w] + \theta \mathcal{H}_4[u_x, w] = 0, \quad (11)$$

with u , θ , σ , and w being the unknowns displacement, absolute temperature, elasto-plastic stress, and freezing index, respectively, ρ , μ , C_V , and ν being positive constants, f , g being given functions, and $\mathcal{H}_1, \dots, \mathcal{H}_4$, and \mathcal{F} being hysteresis operators.

A large-time asymptotic result for this system will be presented. This result has been derived in [1] under the assumption that for all $\varepsilon, w \in C([0, \infty))$ one has

$$(\mathcal{F}_1[\varepsilon, w])_t \leq \varepsilon_t \mathcal{H}_1[\varepsilon, w] + w_t \mathcal{H}_3[\varepsilon, w] \quad \text{a. e. in } (0, \infty),$$

and that a potential \mathcal{F}_2 (see [3]) exists such that an analogous inequality holds for \mathcal{H}_2 and \mathcal{H}_4 . These conditions model the energy dissipation during hysteresis loops. It has been further assumed that $\mathcal{F}_2[\varepsilon, w]$ can be bounded from below by $-C\mathcal{F}_1[\varepsilon, w]$, with some positive constant C and that $\mathcal{H}_1, \dots, \mathcal{H}_4$ satisfy appropriate versions of the outward pointing condition, which will be discussed in the sequel. This outward pointing condition allows to generalize methods that have been applied to derive uniform a priori bounds for solutions to equations involving the superposition with nonlinear functions, and is discussed further in [1,2]. Let $T > 0$ be a final time. Let $\delta > 0$, $h \geq 0$, $A \leq a \leq b \leq B$, be values. A mapping $\mathcal{H} : C([0, T]) \rightarrow C([0, T])$ is said to be *pointing outwards with bound h in the δ -neighbourhood of $[A, B]$ for initial values in $[a, b]$* , if for every $t \in [0, T]$ and every $u \in C[0, T]$ such that

$$u(0) \in [a, b], \quad u(s) \in]A - \delta, B + \delta[\quad \forall s \in [0, t], \quad (12)$$

we have

$$(\mathcal{H}[u](t) - h)(u(t) - B)^+ \geq 0, \quad (\mathcal{H}[u](t) + h)(u(t) - A)^- \leq 0. \quad (13)$$

For such an operator it can be shown that for all $u^0 \in [a, b]$ and all functions $v \in C([0, T])$ with $\|v\|_{C([0, T])} < \delta/2$ holds: If u is a local solution to the initial value problem $u_t(t) + \mathcal{H}[u + v](t) = 0$, $u(0) = u^0$ then u stays within the interval $[A - \delta/2, B + \delta/2]$.

Parts of the presented results are a joined work with P. Krejčí, WIAS, Berlin.

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QUASILINEAR HYPERBOLIC EQUATIONS WITH HYSTERESIS

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We will study a hyperbolic equation of first order of the form

$$u_t + [\phi(u)]_x = 0, \quad u(0) = u_0 \tag{14}$$

and the corresponding quasilinear hyperbolic equation with hysteresis

$$\frac{\partial}{\partial t}(u + w) + \sum_{j=1}^N \frac{\partial}{\partial x_j}(b_j u) + cu = f, \tag{15}$$

where $w = \mathcal{F}(u)$ represents hysteresis.

It is well known that even for ϕ and u_0 smooth, (14) exhibits singularities in a finite time. To be able to continue the solution, one has to pass to a generalized concept of weak solutions where discontinuities are allowed. Weak solutions are in general not uniquely determined by the data, and further physically motivated conditions have to be prescribed. The simplest one is an entropy condition stating that the entropy of the system must be decreasing, generalized by Olejnik (for $N = 1$). A different condition was derived by Kružkov and there are many others. Inspired by Kružkovs

work, Crandall shows that the unique integral solution of (14), constructed by the method of nonlinear semigroups, satisfies an entropy condition derived by Kruřkov. We consider the equation (15) coupled with a generalized play or Prandtl-Ishlinskii operator of play type. It was expected, that the integral solution of (15), for which existence was proved by Visintin using the semigroup approach, and which is unique by construction, fulfils a condition of the type introduced by Kruřkov. We present a solution to this problem. The method enables us to deal with continuous and discontinuous hysteresis as well.

EXISTENCE AND NON-EXISTENCE OF SOLUTIONS TO EVOLUTION INCLUSIONS WITH
NON-INVERTIBLE OPERATORS

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Motivated by non-local phase transition models, we consider an abstract evolution inclusion involving a convex lower semicontinuous potential and a linear elliptic operator with a non-trivial null-space. We derive a condition between the potential, the null-space of the operator, and the initial datum, which is sufficient for the existence of a unique solution. An example shows that if this condition is removed, then even a local solution cannot be expected to exist. (Joint work with P. Colli, E. Rocca, and J. Sprekels).

THE GELATION TRANSITION IN COAGULATION MODELS

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Coagulation models describe the growth of particles or clusters by successive mergers. In some cases, a runaway growth occurs which can be interpreted as the appearance of a new phase, the so-called gelation phenomenon. The aim of this talk is to survey recent mathematical results on this issue for the Smoluchowski and Oort-Hulst-Safronov coagulation models.

INVERSE PROBLEMS FOR PHASE-FIELD MODELS WITH MEMORY

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The identification of kernels in phase-field/phase-separation models with memory is, to my knowledge, a quite new problem. Up to now only very few results are available in literature. The pioneering work regarding this subject is due to F. Colombo. In his paper he considers the problem of recovering a time-dependent memory kernel in a non-conserved phase-field system of Caginalp type (including memory effects). On the same subject there is also a preprint by F. Colombo and D. Guidetti, where the authors are concerned with recovering two unknown kernels. The two quoted papers are both characterized by the fact that the kernel(s) is (are) recovered *locally in time*. For the conserved model there is also a paper by myself where the the memory kernel is still determined *locally in time*.

As far as the more challenging *globally in time* determination of the kernel is concerned, there is a work in progress by E. Rocca, G. Schimperna and myself. At present the determination of the kernel seems to be strictly related to a given *a priori* estimate involving the order parameter and the nonlinear (smooth) function β (coming from the convex part of the free energy potential), such as $\|\beta \circ \chi\|_{L^2(0,T;L^1(\Omega))} \leq M$. This condition would be trivially satisfied if one knew that the phase parameter χ is confined into a bounded interval of the real line. Let me note that this is a physically meaningful condition, indeed χ stands here for the *concentration* of the substance subject to the phase separation process.

The aim of this talk will be the discussion of open identification problems related to *nonlocal* phase separation phenomena. The resulting model can be read as a generalization of the well-known Cahn-Hilliard equation including some convolution kernel which will be allowed to depend on some space variables, in addition to time. In this case the geometric domain Ω will be required to be a non-smooth domain such as a cylinder.

A similar problem will be discussed for some Penrose-Fife model with memory.

Finally, if time permits, the regularization of identification problems with data affected by errors will be dealt with.

 RECTIFIABILITY AND DRIVEN MEAN CURVATURE FLOW

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Rectifiability results for varifolds help to prove convergence for implicit time discretizations of Stefan problems with kinetic undercooling and surface tension.

 WELL-POSEDNESS RESULTS TO A PHASE TRANSITION MODEL
 WITH MICROSCOPIC MOVEMENTS AND ACCELERATIONS

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A recent model for phase transition phenomena was proposed by Michel Frémond. The derivation of the model is based on the consideration that microscopic movements give rise to macroscopic effects. Consider a two-phase substance contained in a domain $\Omega \subset \mathbb{R}^n$, $1 \leq n \leq 3$ and let T be a given final time. The volume fraction of one of the phases is chosen as state quantity and denoted by $\chi = \chi(x, t)$, for $x \in \Omega$ and $t \in]0, T[$. Thus, the order parameter χ satisfies the relation $0 \leq \chi \leq 1$. The absolute temperature $\vartheta = \vartheta(x, t)$ is the other state variable for the thermodynamical system, and it has to be non-negative.

Of course, two relations are needed. The virtual power principle (together with the constitutive laws) provides the phase relation. It must be noted that the power of the acceleration forces is neglected. The main novelty is the following energy balance equation, where the effects of the microscopic movements are taken into account, playing the role of source term

$$(1) \quad c_s \partial_t \vartheta + \frac{L}{\vartheta_c} \theta \partial_t \chi - k \Delta \vartheta = \mu (\partial_t \chi)^2 \quad \text{in } Q := \Omega \times]0, T[.$$

Here c_s , L , ϑ_c , k , and μ are nonnegative physical parameters. The resulting system is highly nonlinear and the solutions to related initial-boundary problems are only local (in time). They become global in the one dimensional framework.

Next, when one has to deal with a fast evolution, the power of the acceleration forces has not to be neglected. The previous balance equation is now coupled with

$$(2) \quad \rho_0 \partial_{tt} \chi + \mu \partial_t \chi + \xi - \nu \Delta \chi + \frac{1}{2} (\chi^3 - \chi) = \frac{L}{\theta_c} (\theta - \theta_c) \text{ in } Q,$$

where ρ_0 and ν are nonnegative physical parameters, while $\xi \in \partial I_{[0, +\infty[}(\partial_t \chi)$ which is the subdifferential of the indicator function of the interval $[0, +\infty[$, when the *irreversible* evolution of the phase change occurs ($\xi = 0$ otherwise). Again, the structure of (2) is not good enough to get a global solution. Hence, it is interesting to investigate the one dimensional framework. By using a fixed point argument and some sharp estimates, the existence and uniqueness of a global solution for a related Cauchy-Neumann problem can be proved.

HYSTERESIS MODELS FOR ELASTIC MATERIALS UNDER FINITE STRAIN

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Many materials display elastic behavior together with inelastic behavior like plasticity, magnetism, damage or phase transformations. The latter behavior can be modelled by an internal variable $z : \Omega \rightarrow Z$ and the elastic deformation is denoted $\varphi : \Omega \rightarrow \mathbb{R}^d$. We consider rate-independent models which are based on the abstract mechanical framework of *generalized standard materials* which relies on a stored energy density and a dissipation potential. This theory can be formulated in exact mathematical terms by using an energy-storage functional $\mathcal{E} : [0, T] \times \mathcal{F} \times \mathcal{Z} \rightarrow \mathbb{R} \cup \{\infty\}$ with

$$\mathcal{E}(t, \varphi, z) = \int_{\Omega} W(x, D\varphi(x), z(x)) + \frac{\alpha}{2} |Dz(x)|^2 dx - \langle \ell(t), \varphi \rangle$$

and a dissipation distance $\mathcal{D} : \mathcal{Z} \times \mathcal{Z} \rightarrow [0, \infty]$, $(z_0, z_1) \mapsto \int_{\Omega} D(z_0(x), z_1(x)) dx$.

A pair $(\varphi, z) : [0, T] \rightarrow \mathcal{F} \times \mathcal{Z}$ is called an *energetical solution*, if for all $t \in [0, T]$ we have

(S) **stability** $\mathcal{E}(t, \varphi(t), z(t)) \leq \mathcal{E}(t, \tilde{\varphi}, \tilde{z}) + \mathcal{D}(z(t), \tilde{z})$ for all $(\tilde{\varphi}, \tilde{z}) \in \mathcal{F} \times \mathcal{Z}$;
 (E) **energy balance**
 $\mathcal{E}(t, \varphi(t), z(t)) + \text{Diss}_{\mathcal{D}}(z, [0, t]) = \mathcal{E}(0, \varphi(0), z(0)) + \int_0^t \partial_t \mathcal{E}(\tau, \varphi(\tau), z(\tau)) d\tau.$

The important feature of the energetical formulation is that it is derivative free and, thus, works nonsmooth and highly nonconvex mechanical systems. In particular, it is applicable to material laws for finite-strain elasticity where $F \mapsto W(x, F, z)$ is polyconvex with $+\infty$ for $\det F \leq 0$. We will present existence results for energetical solutions

(obtained recently with A. Mainik and G. Francfort) and discuss some applications to shape-memory alloys as well as in magneto-striction.

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A. MAINIK and A. MIELKE. Existence results for energetic models for rate-independent systems. *Calculus Variations PDEs*, online since April 2004.

MODELS OF PHASE SEPARATION BASED ON A MICROFORCE BALANCE

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Our aim in this talk is to present some models of Cahn-Hilliard and Allen-Cahn equations based on a microforce balance proposed by M. Gurtin. This microforce balance provides a balance for interactions at a microscopic level, whereas standard forces are associated with macroscopic length scales. In particular, we show how such models can account for elastic and thermal effects.

NONLINEAR PROBLEMS FOR PHASE TRANSITIONS IN ELASTIC SOLIDS

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The main part of the talk is concerned with the existence and uniqueness of solutions to a three-dimensional (3-D) nonlinear thermoelasticity system arising as a model of structural phase transitions in shape memory materials [1] - [4].

The problem consists of the viscoelasticity system with fourth order capillarity-like term coupled with the heat conduction equation with mechanical dissipation. The fourth order term results from the strain gradient-dependence of the underlying Landau-Ginzburg free energy.

The elastic energy is a nonconvex multiple-well function of strain with the shape changing qualitatively with temperature. Under assumptions on the growth of this energy with respect to strain and temperature we prove global in time existence and uniqueness of solutions for large data. The existence proof is based on the parabolic decomposition of the elasticity system and the application of the Leray-Schauder fixed point theorem. The main difficulty

we face comes from the nonlinear coupling of mechanical and thermal effects. To solve the problem we have to derive L_∞ - norm and then Hölder-norm estimate on temperature. In 2-D case this can be accomplished with the help of technical energy estimates and imbedding theorems [3]. In 3-D case such procedure is not sufficient. The goal is achieved by combining the procedure of recursive improvement of energy estimates with De Giorgi method [4]. In the second part of the talk we briefly present recent existence results [5], [6] for different classes of models describing isothermal phase transitions in elastic solids which are due to Fried and Gurtin [7]. The models are based on non-conserved or conserved order parameters and have respectively the form of the Allen-Cahn or Cahn-Hilliard systems coupled with elasticity.

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ENDOCHRONIC, BOUC-WEN AND NON-LINEAR KINEMATIC HARDENING MODELS:
A UNIFYING APPROACH BASED ON GENERALIZED NORMALITY

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An effective way to define the flow rules of plasticity models is the so-called generalized associativity. However, endochronic, Bouc-Wen and non-linear kinematic (NLK) hardening models are usually formulated without using this theoretical framework. In this presentation, the proof that these models are associative in a standard generalized sense is made. As a result, the notion of loading function for endochronic and Bouc-Wen models is shown to be

well-posed and a non-standard description of NLK hardening models is also provided. This approach permits to highlight the strong relationships between these three plasticity models and leads to a general proof of the thermodynamic admissibility of the Bouc-Wen models.

HYSTERESIS IN PLASTICITY AND MAGNETISM: THE BOUC MODEL

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In late sixties R. Bouc introduced a model for plasticity and ferromagnetism which takes into account of memory and hysteresis effects. This model is now widely used by engineers, but a rigorous mathematical analysis is missing. The aim of the talk is to describe its main properties.

A PHASE TRANSITION MODEL WITH THE POSSIBILITY OF VOIDS

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In a joint work with Professor M. Frémond (LCPC, Paris) we deal with phase transition model applied to a two phases system. There is a wide literature on the study of phase transition processes in case that no voids nor overlapping can occur between the two phases. The main novelty of our approach is the possibility of having voids during the phase change. This aspect is described in the model by the mass balance equation whose effects are included by means of the pressure of the system in the dynamical relations. The state variables are the absolute temperature (whose evolution is ruled by the entropy balance equation), the strain tensor (satisfying a quasi-static macroscopic equation), and the volume fractions of the two phases (whose evolutions are described by a vectorial equation coming from the principle of virtual power and related to microscopic motion). The global existence of solutions for the initial-boundary value problem associated to the PDE's system resulting from this model is proved. Uniqueness however is still an open question.

GENERALIZED NEWTONIANS FLOWS WITH PHASE TRANSITIONS

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A modified mathematical model for a binary fluid consists of coupling Cahn-Hilliard equation with a system of equations describing a class of non-Newtonian incompressible flows. The existence of weak solutions for the Cauchy problem is shown for the space dimensions $d = 2$ and 3 . An extension to the phase transition model with non-differentiable free energy is also considered. In spacial dimension $d = 2$, the regularity and the uniqueness of the solutions are shown. This is a joint work with Luisa Consiglieri and Namkwon Kim.

WELL-POSEDNESS RESULTS FOR TWO CLASSES OF GENERALIZED
VISCOUS CAHN-HILLIARD EQUATIONS

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In this talk we will consider two classes of *generalized viscous Cahn-Hilliard equations*, featuring two different laws for the mobility, which is assumed to depend on the chemical potential. Both equations can be framed in the new derivation of equations of Cahn-Hilliard type proposed by M.E. GURTIN. We will present well-posedness and, in one case, regularity results for the systems obtained supplementing each equation with initial and suitable boundary conditions.

WEAK COMPARISON PRINCIPLE OF VISCOSITY SOLUTIONS WITH SHOCKS
FOR SECOND ORDER EQUATIONS

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In [1,2] Giga has introduced the notion of proper viscosity solutions to solve the Cauchy problem for a single nonlinear first order equation of the form

$$(1.1) \quad \partial_t u + H(u, \nabla u) = 0 \quad \text{in } \mathbf{R}^n \times (0, \infty),$$

$$(1.2) \quad u|_{t=0} = u_0 \quad \text{in } \mathbf{R}^n$$

globally-in-time allowing jump discontinuities of solutions. Typical examples include

$$(1.3) \quad \partial_t u - a(u)|\nabla u| = 0,$$

$$(1.4) \quad \partial_t u - b(u)(1 + |\nabla u|^2)^{1/2} = 0,$$

where a and b are not nonincreasing. The conventional theory of viscosity solutions does not apply for such problems including conservation laws. As explained the notion of proper viscosity solution is more restrictive than usual viscosity solution; the proper viscosity solution requires some control on the speed of shocks (jump discontinuities) while the conventional viscosity solution does not require such a control. In [1] he has established various comparison principles for proper viscosity solutions and constructed a unique global proper viscosity solution for various situations.

In my talk we also extend the notion of proper viscosity solutions so that it applies to some second order problems including

$$(1.5) \quad \partial_t u - a(u)|\nabla u| = \sigma|\nabla u| \operatorname{div}(\nabla u/|\nabla u|),$$

$$(1.6) \quad \partial_t u - b(u)(1 + |\nabla u|^2)^{1/2} = \sigma(1 + |\nabla u|^2)^{1/2} \operatorname{div}(\nabla u/(1 + |\nabla u|^2)^{1/2})$$

with $\sigma > 0$. If $\sigma = 0$, (1.5) and (1.6) is nothing but (1.3) and (1.4) respectively. The equation (1.5) requires that each y -level set of u moves by $a(y)$ plus its mean curvature. The equation (1.6) requires that the graph of u moves by $b(y)$ plus its upward mean curvature. The solution of (1.6) may cease to be continuous in a finite time when b is not nonincreasing. Thus the notion of proper solution is expected to be useful to extend the solution for such problems. We can prove weak comparison theorem including equation (1.5), (1.6).

We shall establish a comparison principle for proper viscosity solution solving a second order parabolic equation of the form

$$(2.1) \quad u_t + H(u, \nabla u, \nabla^2 u) = 0 \quad \text{in } \mathbf{R}^n \times (0, \infty),$$

under a periodic boundary condition. We assume that H satisfies the following conditions.

[(H1)]

$H : \mathbb{R} \times \mathbb{R}^n \setminus \{0\} \times \mathbb{S}^n \rightarrow \mathbb{R}$ is continuous.

2. For each $M > 0$ there exists a constant $C = C(M) \in \mathbb{R}$ such that

$$\begin{aligned} H(r, p, S_1 + s_2 p \otimes p) - H(r', p, -T_1 - t_2 p \otimes p) \\ \geq C|r - r'| (K_1 + |p| + 1) \end{aligned}$$

holds for all $r, r' \in \mathbb{R}$ with $|r|, |r'| \leq M$ and $p \in \mathbb{R}^n \setminus \{0\}$, whenever $S_1, T_1 \in \mathbb{S}^n$ and $s_2, t_2 \in \mathbb{R}$ satisfy $\langle S_1 \xi, \xi \rangle + \langle T_1 \eta, \eta \rangle \leq K_1 |\xi - \eta|^2$ and $s_2 a^2 + t_2 b^2 \leq K_2 |a - b|^2$ for all $\xi, \eta \in \mathbb{R}^n$ and $a, b \in \mathbb{R}$ where $K_1, K_2 \in \mathbb{R}_{\geq 0}$ are nonnegative constants independent of ξ, η, a, b .

3. For each $M > 0$ there exists a constant $C' = C'(M) \in \mathbb{R}$ such that

$$H_*(r, 0, 0) - H^*(r', 0, 0) \geq C'|r - r'|$$

holds for all $r, r' \in \mathbb{R}$ with $|r|, |r'| \leq M$.

4. There exists a concave nonnegative function $\alpha(\cdot) : [0, +\infty) \rightarrow \mathbb{R}_{\geq 0}$ satisfying $\lim_{\lambda \downarrow 0} \alpha(\lambda) = 0$ and $\int_0^1 d\lambda/\alpha(\lambda) < +\infty$ and there exists a function $H_\infty : \mathbb{R} \times \mathbb{R}^n \setminus \{0\} \times \mathbb{S}^n \rightarrow \mathbb{R}$ such that

$$\lim_{\lambda \downarrow 0} \sup_{\substack{|r| \leq N \\ |\sigma| \leq N \\ \delta \leq |p| \leq N \\ \|X\| \leq N}} |\lambda H(r, p/\lambda, X/\lambda + \sigma\alpha(\lambda)p \otimes p/\lambda^3) - H_\infty(r, p, X)| = 0$$

for all $\delta, N > 0$ with $\delta \leq N$.

We call u is a proper sub(super)solution of (2.1) in $\mathbb{T}^n \times (0, T)$ with $\mathbb{T}^n := \prod_{i=1}^n (\mathbb{R}/\omega_i\mathbb{Z})$ for $\omega_i > 0$ ($i = 1, 2, \dots, n$) if u is a proper sub(super)solution of (2.1) in $\mathbb{R}^n \times (0, T)$ and u is periodic in all space variables, i.e., $u(x, t) = u(x + \omega_i e_i, t)$ where e_i is the unit vector in the direction of the x_i -axis.

Theorem 0.1 (Weak Comparison Principle). *Assume (H1)-(H4). Let u and v be bounded proper sub- and supersolutions of (2.1) in $\mathbb{T}^n \times (0, T)$, respectively. If $u^*(x, 0) < v_*(x, 0)$ for all $x \in \mathbb{T}^n$, then $u^* < v_*$ in $\mathbb{T}^n \times [0, T)$.*

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MICROGRAVITY ACTION ON PHASE TRANSITIONS PHENOMENA DURING SPACE EXPERIMENTS ON SEMICONDUCTOR CRYSTAL GROWTH

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Growth of high-performance semiconductor crystals in microgravity space conditions may be one of the major directions of development of space high-technology processing. Unfortunately most fulfilled experiments have not given the expected positive results. Research

of structure homogeneity of single crystals grown on-board space vehicles of various types showed that microaccelerations, which take place in the location of electrically heating furnaces determine degree of uniformity of components' distribution along the cross-sections of the crystals. Even in case of specialized satellite "Photon", onboard of which an extremely low level of micro-gravity was registered, the direction of resulting vector of microaccelerations changed in magnitude and moved in various directions during recording. Space experiments showed that the residual microacceleration, being transverse to direction of crystallization and having the value more than 10^{-6} g (g - gravity on Earth surface), causes non-uniform distribution of melt components along cross-sections of crystals grown in space. Effect of small forces on macro-segregation of components during phase transition in space experiments on semiconductor crystal growth was discovered [1] and lately confirmed [2-3]. The impact of residual microaccelerations having ballistic and technical origins could be reduced if: - altitude of spacecraft's orbit is sufficiently high (about 1000 km), - crystal growth furnace is placed in mass-center of the spacecraft and is oriented that velocity vector of directional crystallization has reversed direction to the microaccelerations being lower than 10^{-7} g. However, it is difficult to expect the fulfillment of these requirements onboard large and multipurpose manned space stations. Idea of small satellites using task sharing (microgravity and power supply) seems to be more fruitful. The concept of special high orbiting satellite system including free flyer supplied by wireless power transmission (WPT) from external power source may be possible way to solve problem of growth of semiconductor single-crystals with extraordinary homogeneity of structure.

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LONG TIME BEHAVIOR OF CAGINALP'S MODEL WITH SINGULAR POTENTIAL

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In this talk we will present some recent results, obtained in collaboration with Maurizio Grasselli and Hana Petzeltová, concerning regularity properties and long time behavior of solutions to the so-called phase-field model of Caginalp type. Namely, assigned a smooth and bounded domain $\Omega \subset \mathbb{R}^3$, we consider the following system of equations in $\Omega \times (0, +\infty)$:

$$(\theta + \chi)_t - \Delta \theta = 0, \quad (1)$$

$$\chi_t - \Delta \chi + W'(\chi) = \theta. \quad (2)$$

This system, complemented by the initial conditions, by Dirichlet conditions for the temperature θ , and by no-flux boundary conditions for the order parameter χ , has been introduced by Caginalp in order to describe some non isothermal phase transition processes. Actually, it is well known that under very general hypotheses on the *potential* W such an initial-boundary value problem admits a unique and global in time solution.

Our aim will be to show further properties of this solution under mild assumptions on W , which might in particular be singular as χ approaches the values ± 1 corresponding to the pure states. We can actually prove that any solution $(\theta(t), \chi(t))$ emanating from initial data θ_0, χ_0 of *bounded energy* (i.e. such that $\theta_0 \in L^2(\Omega)$, $\chi_0 \in H^1(\Omega)$, and $W(\chi_0) \in L^1(\Omega)$), as t becomes greater than a critical time $\delta > 0$ depending on the data, gains the $C^\infty(\bar{\Omega})$ regularity and becomes “well-separated” from the barriers $\chi = \pm 1$ in $L^\infty(\Omega)$ -norm.

Moreover, we will show that, under additional assumptions on the restriction of W to $(-1, 1)$, the (whole) trajectory $(\theta(t), \chi(t))$ tends, as $t \nearrow +\infty$, to a *unique* stationary state $(0, \chi_\infty)$, where in particular χ_∞ is a solution of the homogeneous Neumann problem for

$$-\Delta \chi_\infty + W'(\chi_\infty) = 0. \quad (3)$$

We point out that this result is not trivial since the Neumann problem for (3) can have infinitely many solutions under our hypothesis on W .

ON A DOUBLY NONLINEAR MODEL FOR THE EVOLUTION
OF DAMAGING IN VISCOELASTIC MATERIALS

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We consider a model describing the evolution of damage in visco-elastic materials, for which both the stiffness and the viscosity properties degenerate during the damaging. The equations of motion is hyperbolic and degenerating both in the elastic and the viscosity coefficient. The evolution of the damage parameter is described by a doubly nonlinear variational inclusion, due to the presence of two maximal monotone graph involving the phase parameter and its time derivative. Existence of a solution is proved in some subinterval of time in which the damage process is not complete. Uniqueness is established for a Lipschitz approximation of one of the graphs in the variational inclusion for the phase. This is a joint work with Elena Bonetti and Giulio Schimperna (Università di Pavia).

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Let Ω be a bounded domain in R^2 with a smooth boundary, and let σ be a given nonnegative and Lipschitz continuous function on $\bar{\Omega}$.

In this talk, we shall consider steady-state solutions of the following coupled system of two kinetic equations:

$$\begin{cases} (\theta + w)_t - \Delta(\theta + \mu\theta_t) = 0, \\ w_t + \partial V_\sigma(w) + \partial I_{[-1,1]}(w) \ni w + \theta, \end{cases} \quad \text{a.e. in } Q := (0, +\infty) \times \Omega; \quad (1)$$

subject to suitable initial-boundary conditions, and discuss about the stability that they have in the dynamical system generated by (1). This system is a mathematical model to represent the dynamics of solid-liquid phase transitions, and in the context θ is the relative temperature, and w is the nonconserved order parameter.

The first equation is a heat equation with the effect of latent heat. In this equation, the heat flux is settled on the basis of Fourier law, but a time-relaxation term $\mu\theta_t$ (viscosity) is additionally inserted with a (small) positive coefficient μ . Since the viscosity guarantees the smoothness of the velocity of the heat diffusion, we can expect to have more regular estimates for the temperature than that as in the usual heat equation.

The second equation is an Allen-Cahn type equation, and it is derived as the gradient flow of the following functional \mathcal{F}_θ (known as free energy):

$$\mathcal{F}_\theta(w) := V_\sigma(w) + \int_\Omega \left\{ I_{[-1,1]}(w) - \frac{1}{2}w^2 - \theta w \right\} dx, \quad w \in L^2(\Omega).$$

Here, V_σ is the component to characterize the contribution from the interface (interfacial energy), and its mathematical expression is rigorously given by the l.s.c. regularization of the functional $z \in W^{1,1}(\Omega) \cap L^2(\Omega) \mapsto \int_\Omega \sigma |\nabla z| dx$. ∂V_σ is the subdifferential of V_σ in the topology of $L^2(\Omega)$, $I_{[-1,1]}$ and $\partial I_{[-1,1]}$ are respectively the indicator function on the compact interval $[-1, 1]$ and its subdifferential. Formally, the density of the interfacial energy V_σ is minimized on the set $\sigma^{-1}(0)$ of zero-points of σ . So, it is expected that some anisotropic effects of materials would be represented up to the setting of $\sigma^{-1}(0)$.

In this talk, we first see some examples of steady-state solutions, and secondly discuss about their stability in the dynamical system generated by (1). Consequently, it will be shown that some interfaces having crystal-like shapes can be represented by minimizers of free energies in some special settings of σ , and also those minimizers have restoring forces for oscillations of parameters in corresponding dynamical systems.

POROELASTIC FILTRATION COUPLED TO STOKES FLOW

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The formulation and mathematical analysis is presented for a system modeling consolidation and diffusion of a slightly compressible fluid through a partially saturated porous elastic medium coupled to the Stokes flow in an adjacent open channel.

DENSITY ESTIMATES, UNIFORM CONVERGENCE OF LEVEL SETS AND PLANELIKE
MINIMIZERS FOR PHASE TRANSITION MODELS IN PERIODIC MEDIA

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We consider a (possibly singular or degenerate) Ginzburg-Landau type phase-transition model. We prove density estimates for absolute minimizers and we deduce the uniform convergence of level sets and the existence of plane-like minimizers in periodic media. Part of this work has been done in collaboration with A. Petrosyan.
