

Trasformate di Fourier di funzioni.

$u(t)$	$\hat{u}(f) := \int_{-\infty}^{+\infty} u(t)e^{-2\pi i f t} dt$	
$H(t)e^{-at}$	$\frac{1}{a + 2\pi i f}$	$a \in \mathbb{C}, \operatorname{Re} a > 0$
$e^{-a t }$	$\frac{2a}{a^2 + (2\pi f)^2}$	$a \in \mathbb{C}, \operatorname{Re} a > 0$
$\frac{1}{a^2 + t^2}$	$\frac{\pi}{a} e^{-a 2\pi f }$	$a \in \mathbb{C}, \operatorname{Re} a > 0$
$e^{-\pi t^2}$	$e^{-\pi f^2}$	
$\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{t^2}{2\sigma^2}}$	$e^{-2\pi^2\sigma^2 f^2}$	$\sigma \in \mathbb{C}, \arg(\sigma) < \pi/4$
$H(t)t^{\beta-1}e^{-at}$	$\frac{\Gamma(\beta)}{(a + 2\pi i f)^\beta}$	$a, \beta \in \mathbb{C}, \operatorname{Re} a, \operatorname{Re} \beta > 0$
$\frac{1}{(t-a)^n}$	$2\pi i H(-f) \frac{(-2\pi i f)^{n-1}}{(n-1)!} e^{-2\pi i a f}$	$a \in \mathbb{C}, n \in \mathbb{N}, \operatorname{Im} a > 0$
$\frac{1}{(t-a)^n}$	$-2\pi i H(f) \frac{(-2\pi i f)^{n-1}}{(n-1)!} e^{-2\pi i a f}$	$a \in \mathbb{C}, n \in \mathbb{N}, \operatorname{Im} a < 0$
$\operatorname{rect}(t) = \chi_{(-1/2, 1/2)}(t)$	$\operatorname{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$	
$\operatorname{rect}(t/a) = \chi_{(-a/2, a/2)}(t)$	$a \operatorname{sinc}(af)$	$a > 0$
$\chi_{(a,b)}(t)$	$(b-a)e^{-\pi i(a+b)f} \operatorname{sinc}((b-a)f),$	
$\operatorname{sign}(t) \operatorname{rect}(t/a)$	$\frac{1 - \cos(\pi a f)}{\pi i f} = \frac{\sin^2(\frac{\pi}{2} a f)}{\frac{\pi}{2} i f}$	$a > 0$
$\Delta(t) = (1 - t)^+$	$\operatorname{sinc}^2(f) = 2 \frac{1 - \cos(2\pi f)}{(2\pi f)^2}$	$a > 0$
$\frac{1}{(a^2 + t^2)(b^2 + t^2)}$	$\pi \frac{ae^{-b 2\pi f } - be^{-a 2\pi f }}{ab(a^2 - b^2)}$	$a, b > 0, a \neq b$
$\frac{1}{(a^2 + t^2)^2}$	$\frac{\pi}{2a^3} (1 + a 2\pi f) e^{-a 2\pi f }$	$a > 0$
$\frac{1}{a^4 + t^4}$	$\frac{\pi}{a^3} e^{-\sqrt{2\pi a} f } \sin(\pi/4 + \sqrt{2\pi a} f)$	$a > 0$
$\chi_{(-1,1)}(t) \log t $	$-\frac{2}{2\pi f} \operatorname{Si}(2\pi f)$	

Trasformate di Fourier di distribuzioni.

$\langle \hat{u}, \phi \rangle$	$\langle u, \hat{\phi} \rangle$	$u \in \mathcal{S}', \phi \in \mathcal{S}$
δ	1	
$\delta(t - \alpha)$	$e^{-2\pi i \alpha f}$	
$\delta^{(k)}$	$(2\pi i f)^k$	
t^k	$\left(\frac{i}{2\pi}\right)^k \delta^{(k)}$	
$e^{2\pi i \alpha t}$	$\delta(f - \alpha)$	
$\cos(2\pi \alpha t)$	$\frac{1}{2}(\delta(f - \alpha) + \delta(f + \alpha))$	$\alpha \in \mathbb{R}$
$\sin(2\pi \alpha t)$	$\frac{1}{2i}(\delta(f - \alpha) - \delta(f + \alpha))$	$\alpha \in \mathbb{R}$
$\frac{ t ^{\lambda-1}}{\Gamma(\lambda)}$	$2 \cos(\lambda\pi/2) 2\pi f ^{-\lambda}$	$0 < \lambda < 1$
$\frac{1}{\sqrt{t}}$	$\frac{1}{\sqrt{f}}$	
$e^{\pm i \alpha t^2}$	$\sqrt{\frac{\pi}{\alpha}} e^{\mp i \left[\frac{\pi^2 f^2}{\alpha} - \frac{\pi}{4} \right]}$	$\alpha > 0$
v.p. $\frac{1}{t}$	$-i\pi \operatorname{sign}(f)$	
p.f. $\frac{1}{t^n}$	$-i\pi \frac{(-2\pi i f)^{n-1}}{(n-1)!} \operatorname{sign}(f)$	$n \in \mathbb{N}$
$H(t)$	v.p. $\frac{1}{2\pi i f} + \frac{1}{2}\delta$	
$\operatorname{sign}(t)$	v.p. $\frac{1}{\pi i f}$	
$t^n \operatorname{sign}(t)$	p.f. $\frac{2n!}{(2\pi i f)^{n+1}}$	$n \in \mathbb{N}$
$\frac{\sin 2\pi \alpha t}{ t }$	$-i \log \left \frac{f + \alpha}{f - \alpha} \right $	$\alpha \in \mathbb{R}$
$\tilde{\delta}_T = \sum_{k \in \mathbb{Z}} \delta(t - kT)$	$\sum_{k \in \mathbb{Z}} e^{2\pi i k T f} = \frac{1}{T} \tilde{\delta}_{1/T} = \frac{1}{T} \sum_{k \in \mathbb{Z}} \delta(f - k/T)$	$T > 0$

Trasformate di Laplace.

$f(t)$ causale	$F(s) = \mathcal{L}(f)(s) = \int_0^{+\infty} f(t)e^{-st} dt$	
δ	1	
$\delta(t - \alpha)$	$e^{-\alpha s}$	$\alpha \geq 0$
$\delta^{(k)}$	s^k	
$H(t)$	$\frac{1}{s}$	
$H(t)e^{\lambda t} \frac{t^{\alpha-1}}{\Gamma(\alpha)}$	$\frac{1}{(s - \lambda)^\alpha}$	$\operatorname{Re} \alpha > 0, \lambda \in \mathbb{C}$
$H(t) \frac{e^{\alpha t} - e^{\beta t}}{\alpha - \beta}$	$\frac{1}{(s - \alpha)(s - \beta)}$	$\alpha, \beta \in \mathbb{C}, \alpha \neq \beta$
$H(t) \sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\omega \neq 0$
$H(t) \cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\omega \neq 0$
$H(t) \sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$	$\omega \neq 0$
$H(t) \cosh \omega t$	$\frac{s}{s^2 - \omega^2}$	$\omega \neq 0$
$H(t) \log t$	$\frac{\Gamma'(1) - \log s}{s}$	
p.f. $t_+^{-1} = \frac{d}{dt} H(t) \log t$	$\Gamma'(1) - \log s$	
$H(t) e^{-\frac{t^2}{4\sigma^2}}$	$e^{\sigma^2 s^2} \operatorname{erfc}(\sigma s)$	$\sigma > 0$
$H(t) J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$	$a > 0$
$H(t) J_0(2\sqrt{at})$	$\frac{e^{-s/a}}{s}$	$a > 0$

La funzione Γ di Eulero.

$$\begin{aligned}\Gamma(\lambda) &:= \int_0^{+\infty} t^{\lambda-1} e^{-t} dt, \quad \operatorname{Re} \lambda > 0; \\ \Gamma(\lambda + 1) &= \lambda \Gamma(\lambda), \quad \lambda \in \mathbb{C} \setminus \{0, -1, -2, \dots, -n, \dots\} \\ \Gamma(n) &= (n-1)! \quad n = 1, 2, \dots \\ \Gamma\left(\frac{1}{2}\right) &= \sqrt{\pi}, \quad \Gamma(n + 1/2) = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^n} \sqrt{\pi} = \frac{(2n)!}{2^{2n} n!} \sqrt{\pi} \\ \Gamma(\lambda) \Gamma(1-\lambda) &= \frac{\pi}{\sin \pi \lambda} \\ \Gamma'(1) &= -\gamma = \lim_{n \uparrow +\infty} \left(\log n - \sum_{k=1}^n \frac{1}{k} \right)\end{aligned}$$

Error functions

$$\operatorname{erf} t = \frac{2}{\sqrt{\pi}} \int_0^t e^{-s^2} ds, \quad \operatorname{erfc} t = 1 - \operatorname{erf} t = \frac{2}{\sqrt{\pi}} \int_t^{+\infty} e^{-s^2} ds$$

Funzioni di Bessel.

$$\begin{aligned}J_\nu(t) &:= \sum_{n=0}^{+\infty} (x/2)^\nu \frac{(-1)^n (x/2)^{2n}}{n! \Gamma(n + \nu + 1)} \\ J_\nu'' + \frac{1}{t} J_\nu' + \left(1 - \frac{\nu^2}{t^2}\right) J_\nu &= 0\end{aligned}$$