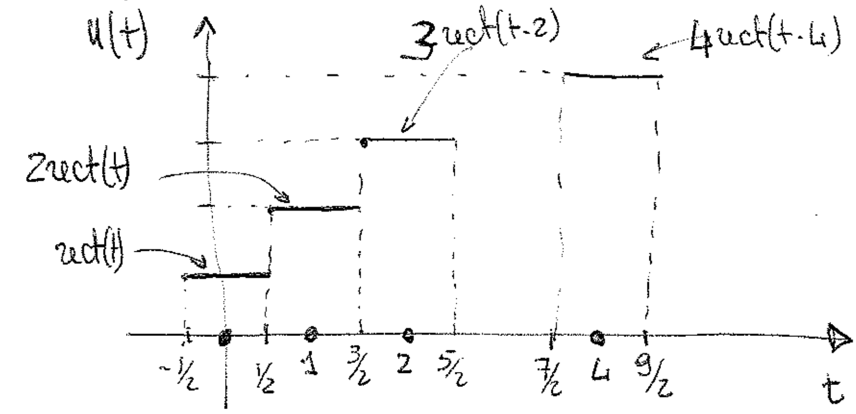
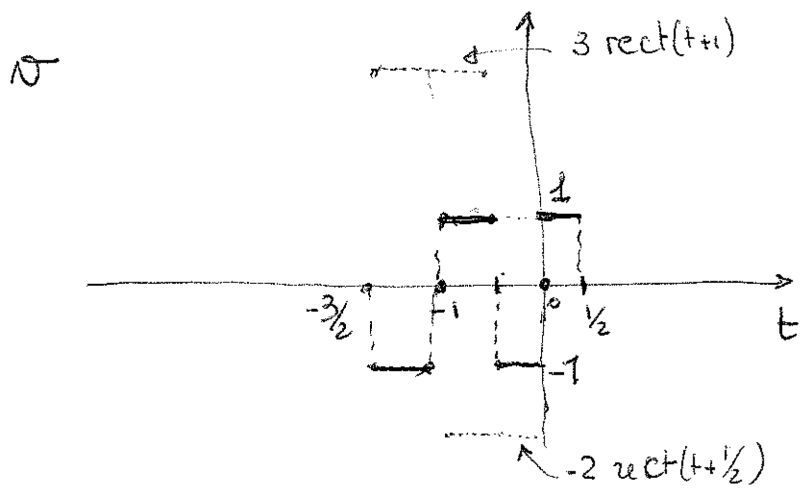
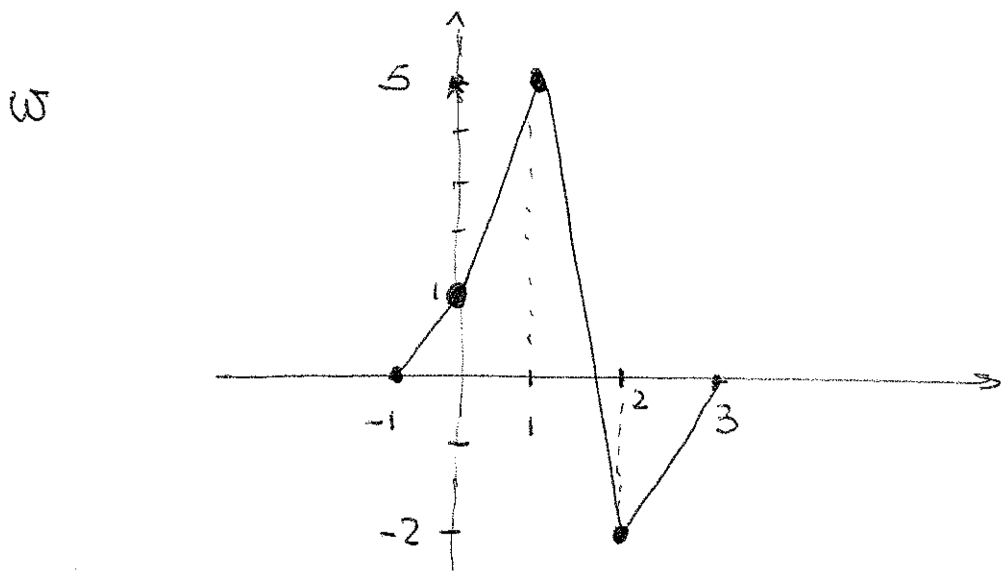
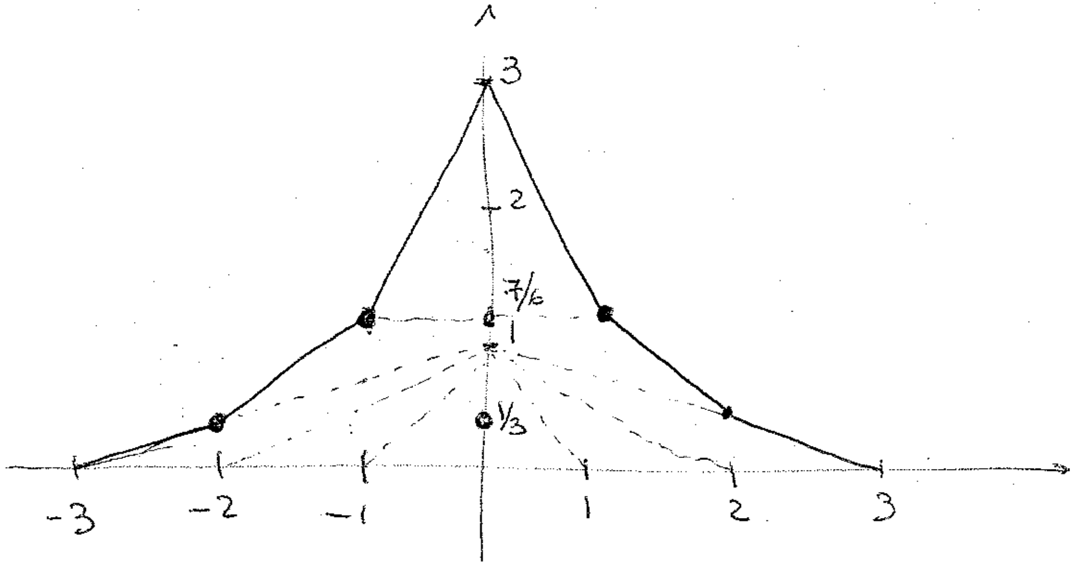
Esercizio 4.1

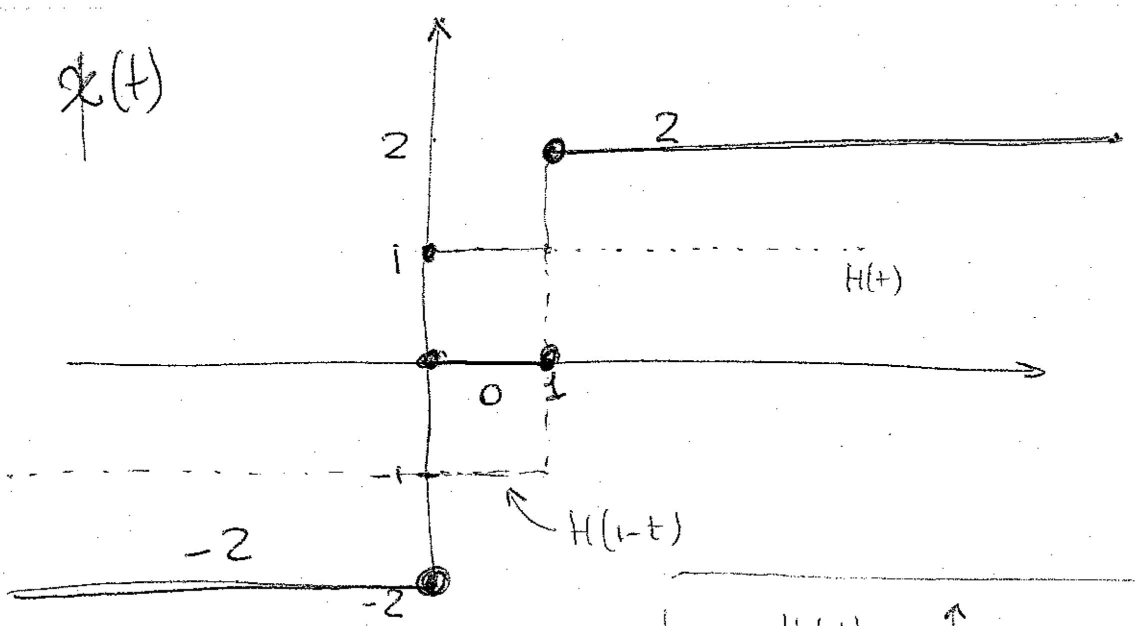


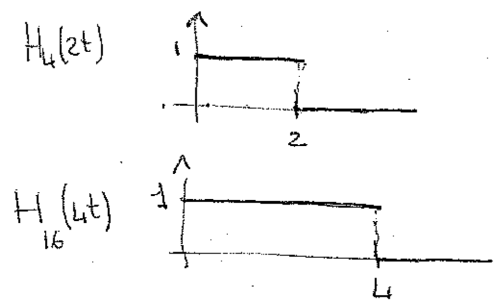


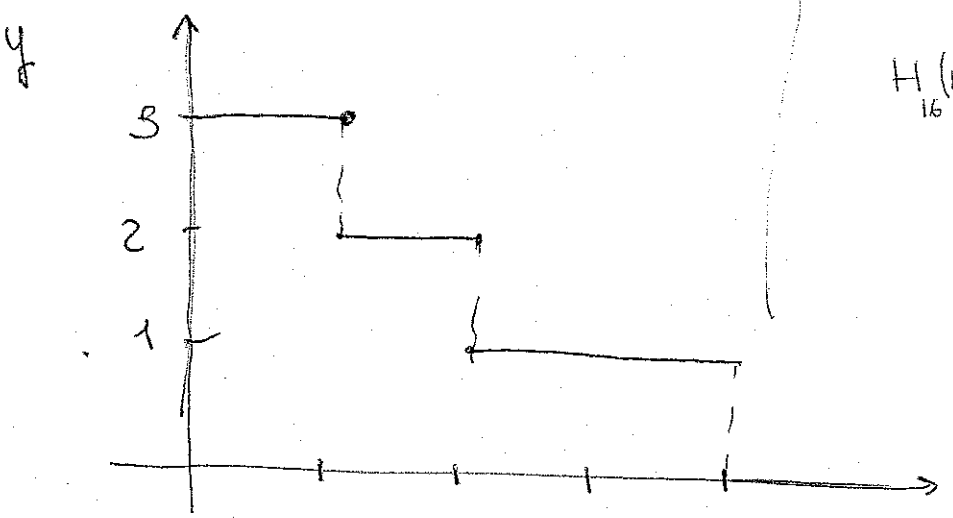


(Sugg. calcolare i valori nei nodi $-1, 0, 1, 2, 3$
e poi interpolare linearmente)


3
2
7/6
1
1/3
-3
-2
-1
1
2
3

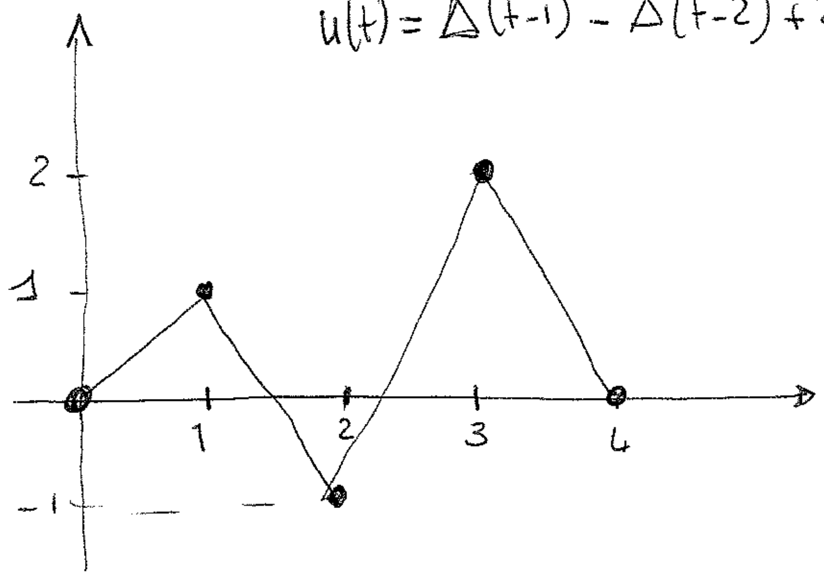

x(t)
2
2
1
H(t)
0
1
H(1-t)
-2
-2


H4(2t)
1
2
H16(4t)
1
4

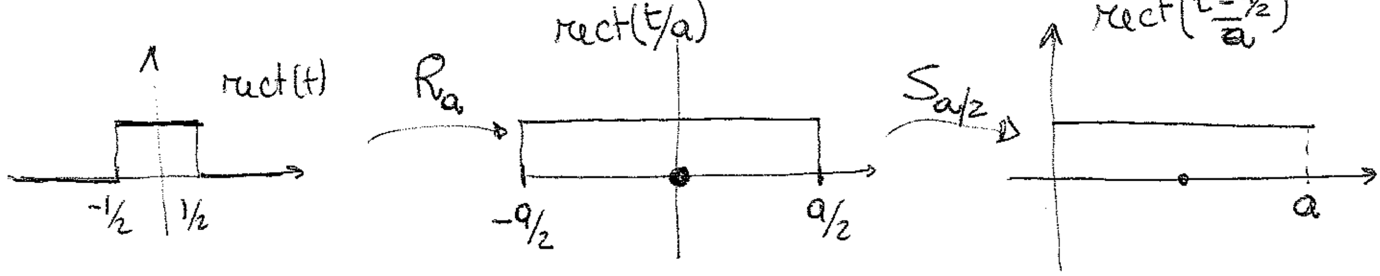

y
3
2
1

Esercizio 4.2

$$u(t) = \Delta(t-1) - \Delta(t-2) + 2\Delta(t-3)$$



Esercizio 4.3



$$H_a(t) = \mathrm{rect}(t/a - 1/2)$$

$$\chi_{a,b} = H_b - H_a \qquad (\text{se } 0 < a < b)$$

$$\chi_{a,b}(t) = H_b(t) + H_a(-t) \qquad (\text{se } a < 0 < b)$$

$$\chi_{a,b}(t) = H_a(-t) - H_b(-t) \qquad (\text{se } a < b < 0)$$

$$\Delta(t) = (1-t)H(t) + (1+t)H(-t) + tH(t-1) - tH(-t-1)$$

$$|t| = tH(t) - tH(-t) \quad , \quad t^+ = tH(t) \, , \; \mathrm{sign}(t) = H(t) - H(-t)$$

$$\mathrm{sign}(t)\,\mathrm{rect}(t) = \mathrm{rect}(2t - 1/2) - \mathrm{rect}(2t + 1/2)$$

Esercizio 4.4

$u(t) = \rho\cos(t+\theta)$, $\rho = \sqrt{8}$ $\quad \theta = \pi/4$

$v(t) \longrightarrow \quad c = \sqrt{3} - i \quad |c| = \rho = \sqrt{3+1} = 2 \quad \theta = \operatorname{Arg} c = -\pi/6$

$w(t) \longrightarrow \quad c = -2\sqrt{3} + i6\sqrt{3} \quad |c| = \sqrt{3}\sqrt{40} = 2\sqrt{30}$, $\theta = \arccos\left(-\frac{1}{\sqrt{10}}\right)$

Esercizio 4.5

$$\int_0^1 t\sin(2\pi t)\,dt = -\underbrace{\int_0^1 -\frac{\cos 2\pi t}{2\pi}\,dt}_{0} + \left[t\,\frac{-\cos(2\pi t)}{2\pi}\right]_0^1 =$$

$= -\frac{1}{2\pi}$ (ricordare che l'integrale di ogni funzione trigonometrica su un periodo è nullo!)

$$\int_0^2 \cos^2(5\pi t)\,dt =$$

$\cos(5\pi t)$ ha periodo $\frac{2}{5}$, cioè $\frac{1}{5}$ dell'intervallo, quindi l'integrale è pari a 5 volte l'integrale su un periodo

$$= 5\int_0^{2/5}\cos^2(5\pi t)\,dt = 5\cdot\frac{2}{5}\cdot\boxed{\frac{1}{5/2}\int_0^{2/5}\cos^2(5\pi t)\,dt}$$ ← potenza di $\cos(5\pi t) \rightarrow \frac{1}{2}$

$= 5\cdot\frac{2}{5}\cdot\frac{1}{2} = \boxed{1}$

Altro metodo: 5π è multiplo intero di π (pulsazione fond. relativa al periodo 2 - Quindi $\int_0^2\cos^2(5\pi t)\,dt = \underset{\text{periodo}}{2}\cdot\underset{\text{potenza}}{\frac{1}{2}} = 1$

$$\int_0^{12\pi}\sin^2(t)\,dt = \underset{\#\text{ periodi}}{6}\cdot\underset{\text{lungh. periodo}}{2\pi}\cdot\underset{\text{potenza su un periodo}}{\frac{1}{2}} = 6\pi$$

$(\cos t \,|\, \sin t) = 0 \qquad (\cos t, \sin t)$ sono ortogonali

$\mathbb{P}[\cos(3t) + \sin(3t)] = \frac{1}{2} + \frac{1}{2} = 1$

$\langle 1 + \sin(7\pi t)\rangle = 1 \qquad \frac{1}{2\pi}\int_0^{2\pi}\cos^2(12\pi t) + 4\sin^2(24\pi t)\,dt = \frac{1}{2} + 4\cdot\frac{1}{2} = 5/2$ -

Esercizio 4.6

Si

$$\sin^4(\pi t) = \left(\frac{e^{i\pi t} - e^{-i\pi t}}{2i}\right)^4 = \frac{1}{16}\left[e^{i4\pi t} - 4e^{i3\pi t - i\pi t} + 6e^{2i\pi t - 2i\pi t} - 4e^{i\pi t - 3i\pi t} + e^{-i4\pi t}\right]$$

$$= \frac{1}{16}\left[e^{i4\pi t} - 4e^{2\pi i t} + 6 - 4e^{-2\pi i t} + e^{-i4\pi t}\right]$$

sono tutte funzioni 1-periodiche da integrare su 2 periodi

Gli esponenziali si annullano e rimane solo 6

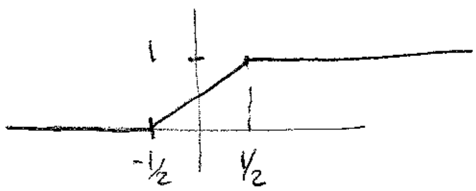
$$\int_0^2 \sin^4(\pi t)\,dt = \frac{12}{16} = \frac{3}{4}$$

$$\int_0^{\pi} \cos^3(2t)\,dt = 0$$

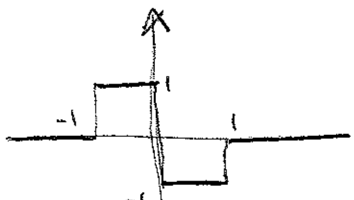
$$\int_0^1 \sin(2\pi t)\sin(4\pi t)\,dt = 0$$ (sono funzioni 1-periodiche - si applica l'ortogonalità).

Esercizio 4.7

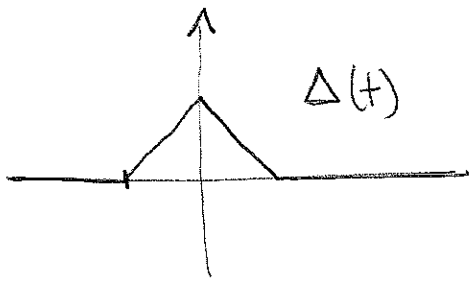
$\mathrm{rect}(t) \rightarrow$



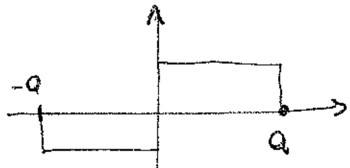
$H_1(-t) * H_1(t)$

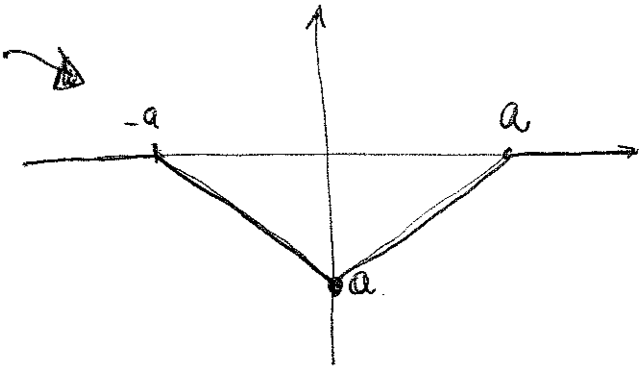


$\rightarrow$

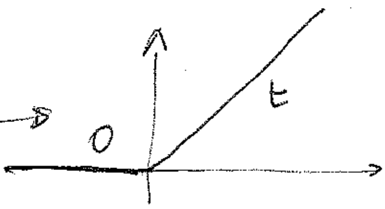


$H_a(-t) - H_a(-t)$

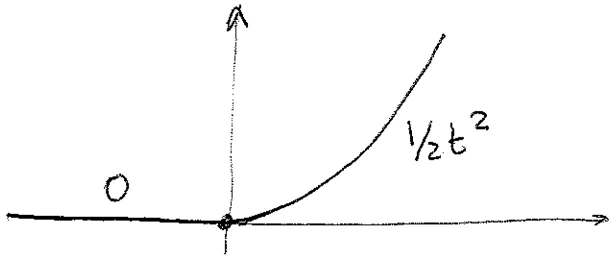




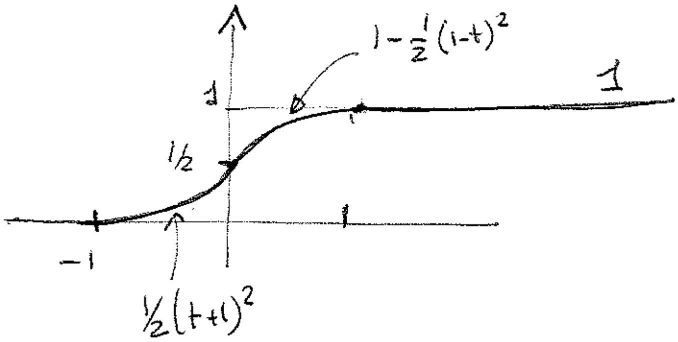
$H(t) \rightarrow$



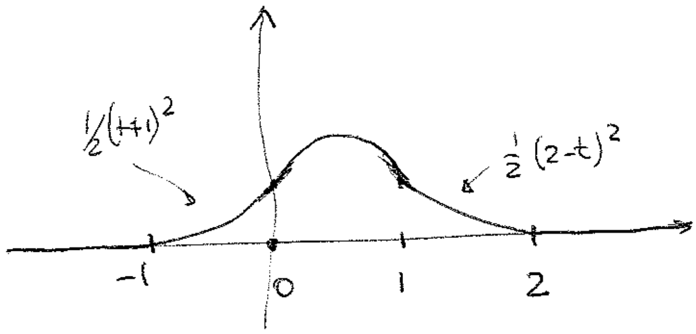
$t\,H(t) \longrightarrow$



$\Delta(t) \longrightarrow$



$\Delta(t)-\Delta(t-1)$



Esercizio 4.8

$|t| \to$ pari $\quad H(t)-H(-t) \to$ dispari, $\text{rect}(t)+\text{rect}(2t)+\text{rect}(3t) \to$ pari

$\Delta(t)-\Delta(t-1)-\Delta(t+1) \to$ pari $\quad \cos(t)\,\text{sign}(t) \to$ dispari

$t^2 \sin(\pi t) \to$ dispari $\quad (t^3-t)\,t^2 \to$ dispari

$\cos(2t)\sin^2(4t) \to$ pari $\quad H_2(t-2)-\text{rect}\left(t/2+3/2\right) \to$ dispari

$t-1 \to$ né pari, né dispari.