

Strong A_∞ weights and quasilinear degenerate elliptic equations

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Abstract

The aim of the talk is to show regularity results of the weak solutions for degenerate elliptic equations of the following kind

$$\operatorname{div} A(x, u(x), \nabla u(x)) + B(x, u(x), \nabla u(x)) = 0$$

under the structure conditions

$$\begin{cases} |A(x, u, \xi)| \leq a\omega(x)|\xi|^{p-1} + b|u|^{p-1} + e \\ |B(x, u, \xi)| \leq c|\xi|^{p-1} + d|u|^{p-1} + f \\ \xi \cdot A(x, u, \xi) \geq \omega(x)|\xi|^p - d|u|^p - g. \end{cases}$$

Here v is a strong A_∞ weight and $\omega = v^{1-\frac{p}{n}}$, $1 < p < n$.

The novelty here is the degeneracy condition given by choice of the weight ω to be a power of a strong A_∞ weight.

Moreover, we assume very mild integrability conditions on the lower order terms and known term. These conditions are sharp and, at least in some instances, are necessary and sufficient.

Our main result is the Harnack inequality for nonnegative weak solutions of the equation under investigation. As a consequence we get some regularity results for the weak solutions.