Macroscopic modeling of Magnetic Shape Memory Alloys

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In the last decade a new class of materials called magnetic shape memory alloys (MSMAs) has been intensively investigated. MSMAs are metallic alloys presenting the superlastic and shape memory effects along with a *giant* magnetostrictive behavior (up to 5-8%) which is the effect of the ferromagnetic nature of martensites. In particular, the martensitic phase in MSMAs presents the classical ferromagnetic texture of magnetic domains. This mesostructure can be modified by magnetic domain wall motion, magnetization rotation, and martensitic variant reorientation. We are focusing here on the *magnetically uniaxial* case, that is to say that martensites are assumed to present just one magnetic *easy axis*. This is specifically the case of cubic-to-tetragonal martensitic transformations (as in Ni_2MnGa, FePd, FePt among others).

We aim at presenting a novel 3D description of the constitutive response of MSMAs as the effect of changes in the internal magnetic field $\boldsymbol{H}$, the total strain $\boldsymbol{\sigma}$, and the absolute temperature T. The linearized (small) strain $\boldsymbol{\varepsilon}$ is additively decomposed into $\boldsymbol{\varepsilon} = \mathbb{C}^{-1}{:}\boldsymbol{\sigma}+\boldsymbol{z}$ where $\mathbb{C}$ is the elasticity tensor whereas $\boldsymbol{z} \in \mathbb{R}^{3\times 3}_{\mathrm{dev}}$ is the inelastic (or transformation) part and shall be regarded as the descriptor of the martensitic structure of the material. Given variant $\boldsymbol{z}$, we obtain the corresponding (directed) easy axis as

$$\mathcal{A}\boldsymbol{z} := \frac{1}{3} m_{\mathrm{sat}} (1,1,1)^{\top} + \mathbb{A}{:}\boldsymbol{z}$$

where m_{sat} is the saturation magnetization, $\mathbb{A}$ is a 3-tensor of components

$$\mathbb{A}_{iii} = -\frac{2}{3}\sqrt{\frac{2}{3}}\frac{m_{\mathrm{sat}}}{\varepsilon_L}, \quad \mathbb{A}_{ijj} = \frac{1}{3}\sqrt{\frac{2}{3}}\frac{m_{\mathrm{sat}}}{\varepsilon_L}, \quad \mathbb{A}_{ijk} = 0$$

for $i,j,k = 1,2,3,\ i \neq j \neq k,$, and ε_L is the maximal strain modulus due to alignment of martensitic variants. In particular, we have that $\mathcal{A}\boldsymbol{z}_i = m_{\mathrm{sat}}\boldsymbol{e}_i$ (coordinate vector) for all pure variants $\boldsymbol{z}_i$ obtained by compression along the respective coordinate directions. A second internal variable is the local (signed) proportion $\alpha \in [-1,1]$ of magnetic domains oriented in the direction of the (directed) easy axis. Additional modeling details and motivation are reported in [1] and the reader is referred to [2, 3] for some relevant modeling discussions.

Our main modeling ansatz is that of directly connecting magnetic and mechanical variables by prescribing the magnetization $\boldsymbol{M}$ of the material in terms of $\boldsymbol{z}$ and α as

$$\boldsymbol{M} = \alpha\mathcal{A}\boldsymbol{z}.$$

In particular, we assume that the magnetic anisotropy of the material is sufficiently strong so that the magnetization stays rigidly attached to the easy axes of the martensitic variants and no magnetization rotation occurs [5]. Note that the specific form of $\mathcal{A}$ is compatible with material symmetries and yields the natural constraint $|\boldsymbol{M}| \leq m_{\mathrm{sat}}$.

We prescribe the Gibbs free energy density of the material (of a constant and normalized density) as

$$G(\boldsymbol{\sigma}, \boldsymbol{H}, T, \boldsymbol{z}, \alpha) := -\frac{1}{2}\boldsymbol{\sigma}:\mathbb{C}^{-1}:\boldsymbol{\sigma} - \boldsymbol{\sigma}:\boldsymbol{z} + \beta(T)|\boldsymbol{z}| + \frac{h}{2}|\boldsymbol{z}|^2 + I_{\varepsilon_L}(\boldsymbol{z})$$
$$+ \frac{1}{2\delta}\alpha^2 + I_{[-1,1]}(\alpha) - \mu_0 \boldsymbol{H}\cdot\alpha\mathcal{A}\boldsymbol{z}. \tag{1}$$

The first line in (1) is exactly the Gibbs energy of the well-known *Souza-Auricchio model* for non-magnetic SMAs [4]. In particular, $T \mapsto \beta(T) \geq 0$ is a specific function of the temperature, $h > 0$ is a hardening modulus, and I_{ε_L} is the *indicator function* of the ball $\{\boldsymbol{z} \in \mathbb{R}^{3\times3}_{\text{dev}} \ : \ |\boldsymbol{z}| \leq \varepsilon_L\}$.

The second line in (1) describes the magnetic behavior of the material. The term $-\mu_0 \boldsymbol{H}\cdot\alpha\mathcal{A}\boldsymbol{z}$ is nothing but the classical *Zeeman energy* term. Note that $\boldsymbol{H}$ stands here for the *internal* magnetic field. In particular, $\boldsymbol{H}$ corresponds to the sum of the applied external field and the corresponding induced demagnetization field. The indicator function $I_{[-1,1]}$ is constraining the scalar α to take values in $[-1, 1]$ and $1/\delta$ is a hardening parameter.

As for the dissipative character of the model, we assume that the inelastic strain $\boldsymbol{z}$ dissipates energy whereas the variable α is non-dissipative. This is of course disputable as the dissipation in α is, for instance, the basic dissipative mechanism in ferromagnetic materials. Still, experiments show that, at small strains, the dissipation in α is negligible with respect to that in $\boldsymbol{z}$. Eventually, the dissipation function associated with $\boldsymbol{z}$ is given by

$$D(\dot{\boldsymbol{z}}) = \sup\left\{\boldsymbol{g}:\dot{\boldsymbol{z}} \ \mid F(\boldsymbol{g}) \leq 0\right\} = \begin{cases} R|\dot{\boldsymbol{z}}| & \text{if } \dot{\boldsymbol{z}} \in \mathbb{R}^{3\times3}_{\text{dev}} \\ \infty & \text{else} \end{cases}$$

Moving from these considerations, the internal variable α can be directly obtained as a function of $\boldsymbol{H}$ and $\boldsymbol{z}$ as $\alpha = \max\big\{-1, \min\big\{1, \delta\mu_0\boldsymbol{H}\cdot\mathcal{A}^*\boldsymbol{z}\big\}\big\}$.

We have proved existence of *energetic solutions* for both the constitutive relation problem and the three-dimensional quasi-static evolution problem ($\boldsymbol{H}$ given). Moreover, we have checked the reduction of this model to the Souza-Auricchio non-magnetic one by means of a rigorous Γ-convergence argument.

References

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